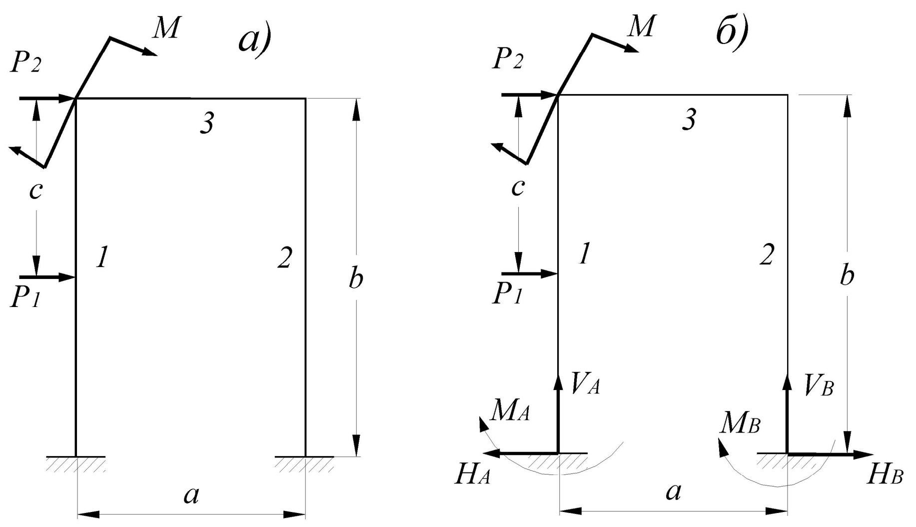
ÐÀÑ×ÅÒ ÏÐÎÑÒÅÉØÈÕ ÑÒÀÒÈ×ÅÑÊÈ ÍÅÎÏÐÅÄÅËÈÌÛÕ ÑÈÑÒÅÌ ÏÐÈ ÈÇÃÈÁÅ

Â ðàçäåëå ðàññìàòðèâàåòñÿ ðàñ÷åò ïðîñòåéøèõ ñòàòè÷åñêè íåîïðåäåëèìûõ ðàì è áàëîê, òàê êàê èìåííî òàêèå ñèñòåìû íàèáîëåå ÷àñòî âñòðå÷àþòñÿ â êîíñòðóêöèÿõ ìàøèí è ìåõàíèçìîâ.

Â ïåðâîé ãëàâå äàííîãî êóðñà óæå ïðèõîäèëîñü âñòðå÷àòüñÿ ñ ðàñ÷åòîì ñèñòåì, äëÿ îïðåäåëåíèÿ óñèëèé â êîòîðûõ íåäîñòàòî÷íî îäíèõ ëèøü óðàâíåíèé ðàâíîâåñèÿ ñòàòèêè, à íåîáõîäèìî ñîñòàâëÿòü äîïîëíèòåëüíûå óðàâíåíèÿ, êîòîðûå íàçûâàþòñÿ óðàâíåíèÿìè ñîâìåñòíîñòè äåôîðìàöèé. Òàêèå ñèñòåìû íàçûâàþò *ñòàòè÷åñêè íåîïðåäåëèìûìè*.

1. Расчет статически неопределимых рам с помощью метода сил

Ðàñ÷åò ñòàòè÷åñêè íåîïðåäåëèìûõ ðàì (Ðèñ. 1) ìåòîäîì ñèë ïðîâîäèòñÿ â ñëåäóþùåé ïîñëåäîâàòåëüíîñòè:



Ðèñ. 1

1. Âû÷åð÷èâàþò ðàñ÷åòíóþ ñõåìó ðàìû, íàïðèìåð, ðàñ÷åòíóþ ñõåìó øïðåíãåëÿ êóëüòèâàòîðà–ðàñòåíèåïèòàòåëÿ;

2. Ïîêàçûâàþò íà ÷åðòåæå âîçìîæíûå îïîðíûå ðåàêöèè;

3. Îïðåäåëÿþò ñòåïåíü ñòàòè÷åñêîé íåîïðåäåëèìîñòè ðàìû. Ñòåïåíü ñòàòè÷åñêîé íåîïðåäåëèìîñòè ðàâíà ÷èñëó "ëèøíèõ" ñâÿçåé, óäàëåíèå êîòîðûõ ïðåâðàùàåò ñòàòè÷åñêè íåîïðåäåëèìóþ ñèñòåìó â îïðåäåëèìóþ ãåîìåòðè÷åñêè íåèçìåíÿåìóþ ñèñòåìó. *Ãåîìåòðè÷åñêè íåèçìåíÿåìîé íàçûâàåòñÿ ñèñòåìà, èçìåíåíèå ôîðìû êîòîðîé âîçìîæíî ëèøü â ñâÿçè ñ äåôîðìàöèåé åå ýëåìåíòîâ.* Ïðè ýòîì ñòàòè÷åñêè îïðåäåëèìàÿ ñèñòåìà íå èìååò íè îäíîé ëèøíåé ñâÿçè. Óäàëåíèå õîòÿ áû îäíîé ñâÿçè ïðåâðàùàåò åå â ãåîìåòðè÷åñêè èçìåíÿåìóþ ñèñòåìó, òî åñòü â *ìåõàíèçì.* ×èñëî "ëèøíèõ" ñâÿçåé îïðåäåëÿþò ïî ôîðìóëå:

$$n = m - k,$$

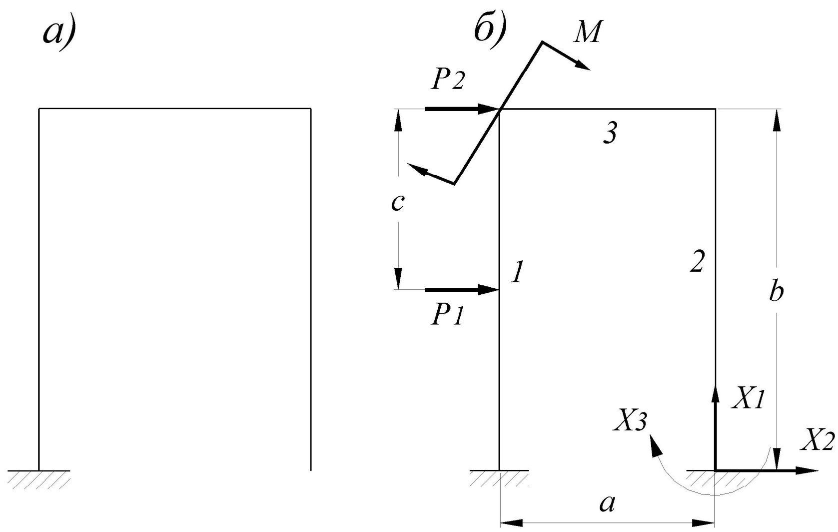
ãäå m– ÷èñëî îïîðíûõ ðåàêöèé:
k– ÷èñëî âîçìîæíûõ óðàâíåíèé ðàâíîâåñèÿ.

Òîãäà äëÿ ïðèíÿòîé ðàìû èìååì $n = 6 - 3 = 3$.

4. Ïðèíèìàþò îñíîâíóþ ñèñòåìó ðàìû. Äëÿ ýòîãî íàäî îòáðîñèòü "ëèøíèå" ñâÿçè, ÷èñëî êîòîðûõ äîëæíî áûòü ðàâíî ñòåïåíè ñòàòè÷åñêîé íåîïðåäåëèìîñòè. Â íàøåì ñëó÷àå ìîæíî óáðàòü, íàïðèìåð, ëåâóþ çàäåëêó. Äëÿ êàæäîé ñòàòè÷åñêè íåîïðåäåëèìîé ñèñòåìû ìîæíî ïîäîáðàòü, êàê ïðàâèëî, íåñêîëüêî îñíîâíûõ ñèñòåì. Òàê, â ïðèíÿòîé ðàìå ìîæíî óáðàòü ïðàâóþ îïîðó è ò.ä. Òîãäà ìû ïîëó÷èì ñòàòè÷åñêè îïðåäåëèìóþ ðàìó, èçîáðàæåííóþ íà ðèñóíêå (Ðèñ. 2, à).

5. Ïðèíèìàþò ýêâèâàëåíòíóþ ñèñòåìó (Ðèñ. 2, á). Äëÿ ýòîãî îòáðîøåííûå ñâÿçè çàìåíÿþò ñèëàìè è ìîìåíòàìè. Âåëè÷èíà èõ äîëæíà áûòü ïîäîáðàíà òàê, ÷òîáû ïåðåìåùåíèÿ ñîîòâåòñòâîâàëè òåì îãðàíè÷åíèÿì, êîòîðûå íàêëàäûâàþòñÿ íà ñèñòåìó îòáðîøåííûìè ñâÿçÿìè. Òî åñòü, ïðàâûé êîíåö ðàìû íå äîëæåí

ïîâîðà÷èâàòüñÿ, èìåòü âåðòèêàëüíîå è ãîðèçîíòàëüíîå ïåðåìåùåíèÿ. Ïðè ýòîì íåèçâåñòíûìè îêàçûâàþòñÿ ñèëû. Îòñþäà è íàçâàíèå *"ìåòîä ñèë"*. Ê îñíîâíîé ñèñòåìå ïðèêëàäûâàþòñÿ è çàäàííûå âíåøíèå ñèëû.



Ðèñ. 2

6. Ñîñòàâëÿþò äåôîðìàöèîííûå óðàâíåíèÿ. Î÷åâèäíî, ÷òî èõ ÷èñëî äîëæíî áûòü ðàâíî ñòåïåíè ñòàòè÷åñêîé íåîïðåäåëèìîñòè. Â íàøåì ñëó÷àå èõ äîëæíî áûòü òðè. Îáîçíà÷èì ïåðåìåùåíèÿ ïðàâîãî êîíöà ðàìû â íàïðàâëåíèÿõ íåèçâåñòíûõ ñèë X_1, $Õ_2$ è $Õ_3$ ñîîòâåòñòâåííî Δ_1, Δ_2 è Δ_3. Òàêèì îáðàçîì, èíäåêñû ïåðåìåùåíèé óêàçûâàþò íà èõ íàïðàâëåíèå. Êàæäîå èç ýòèõ ïåðåìåùåíèé ñêëàäûâàåòñÿ èç ïåðåìåùåíèé îò äåéñòâèÿ ñèë X_1, X_2, $Õ_3$ è âíåøíèõ íàãðóçîê è ðàâíî íóëþ èç–çà îãðàíè÷åíèé, êîòîðûå áûëè íàëîæåíû îòáðîøåííûìè ñâÿçÿìè. Òîãäà ìîæåì çàïèñàòü ñèñòåìó èç òðåõ óðàâíåíèé:

$$\Delta_1 = \Delta_{11} + \Delta_{12} + \Delta_{13} + \Delta_{1Ð} = 0;$$

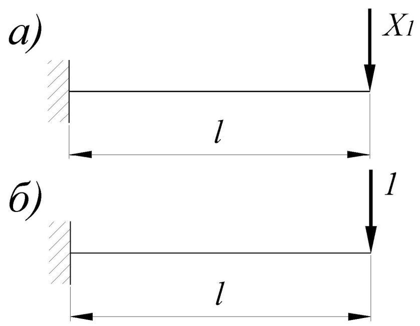
$$\Delta_2 = \Delta_{21} + \Delta_{22} + \Delta_{23} + \Delta_{2Ð} = 0;$$

$$\Delta_3 = \Delta_{31} + \Delta_{32} + \Delta_{33} + \Delta_{3Ð} = 0,$$

ãäå Δ_{11}– ïåðåìåùåíèå ïðàâîãî êîíöà ðàìû â âåðòèêàëüíîì íàïðàâëåíèè îò äåéñòâèÿ ñèëû X_1;

D_{12}– ïåðåìåùåíèå ïðàâîãî êîíöà ðàìû â âåðòèêàëüíîì íàïðàâëåíèè îò äåéñòâèÿ ñèëû $Õ_2$;
D_{13}– ïåðåìåùåíèå ïðàâîãî êîíöà ðàìû â âåðòèêàëüíîì íàïðàâëåíèè îò äåéñòâèÿ ñèëû $Õ_3$;
$D_{1Ð}$– ïåðåìåùåíèå ïðàâîãî êîíöà ðàìû â âåðòèêàëüíîì íàïðàâëåíèè îò äåéñòâèÿ âíåøíèõ ñèë;
D_{21}– ïåðåìåùåíèå ïðàâîãî êîíöà ðàìû â ãîðèçîíòàëüíîì íàïðàâëåíèè îò äåéñòâèÿ ñèëû X_1;
D_{22}– ïåðåìåùåíèå ïðàâîãî êîíöà ðàìû â ãîðèçîíòàëüíîì íàïðàâëåíèè îò äåéñòâèÿ ñèëû X_2;
D_{33}– ïåðåìåùåíèå ïðàâîãî êîíöà ðàìû â ãîðèçîíòàëüíîì íàïðàâëåíèè îò äåéñòâèÿ ñèëû $Õ_3$;
$D_{2Ð}$ – ïåðåìåùåíèå ïðàâîãî êîíöà ðàìû â ãîðèçîíòàëüíîì íàïðàâëåíèè îò äåéñòâèÿ âíåøíèõ ñèë;
D_{31}– êðóãîâîå ïåðåìåùåíèå ïðàâîãî êîíöà ðàìû îò äåéñòâèÿ ñèëû X_1;
D_{32}– êðóãîâîå ïåðåìåùåíèå ïðàâîãî êîíöà ðàìû îò äåéñòâèÿ ñèëû X_2;
D_{33}– êðóãîâîå ïåðåìåùåíèå ïðàâîãî êîíöà ðàìû îò äåéñòâèÿ ñèëû $Õ_3$;
D_{3P}– êðóãîâîå ïåðåìåùåíèå ïðàâîãî êîíöà ðàìû îò äåéñòâèÿ âíåøíèõ ñèë.

Îïðåäåëèì âåðòèêàëüíîå ïåðåìåùåíèå D_{11} êîíöà êîíñîëüíîé áàëêè îò äåéñòâèÿ ñèëû X_1 (Ðèñ. 3).



Ðèñ. 3

Ïîäîáíîå ïåðåìåùåíèå, íî îò ñèëû Ð áûëî îïðåäåëåíî ðàíåå ðàçíûìè ñïîñîáàìè è ðàâíÿëîñü $Pl^3/3EI$. Â íàøåì ñëó÷àå $D_{11}=õ_1l^3/3EI$.

Ïðèëîæèì âìåñòî ñèëû Ð åäèíè÷íóþ ñèëó è îïðåäåëèì âûçâàííîå åþ âåðòèêàëüíîå ïåðåìåùåíèå d_{11}

$$d_{11} = (1 \times l^3)/EI.$$

Ñðàâíèâàÿ ýòó ôîðìóëó ñ ôîðìóëîé, ïîëó÷åííîé äëÿ D_{11}, ìîæåì çàïèñàòü $D_{11} = X_1d_{11}$.

Àíàëîãè÷íî D_{12} = $Õ_2 d_{12}$, D_{31} = $Õ_3 d_{13}$, D_{23} = $Õ_3 d_{23}$ è ò.ä. Òîãäà ñèñòåìà çàïèøåòñÿ ñëåäóþùèì îáðàçîì:

$$D_1 = X_1 d_{11} + Õ_2 d_{12} + Õ_3 d_{13} + D_{1P} = 0;$$
$$D_2 = X_1 d_{21} + Õ_2 d_{22} + Õ_3 d_{23} + D_{2P} = 0;$$
$$D_3 = X_1 d_{31} + Õ_2 d_{32} + Õ_3 d_{33} + D_{3P} = 0.$$

Â îáùåì ñëó÷àå ëþáîå èç ýòèõ óðàâíåíèé ìîæíî íàïèñàòü â ñëåäóþùåì âèäå:

$D_i = X_1 d_{i1} + X_2 d_{i2} + Õ_3 d_{i3} + \ldots + X_n d_{in} + D_{1P} = 0$, ãäå ïåðâûé èç êàæäîãî äâîéíîãî èíäåêñà îçíà÷àåò íàïðàâëåíèå ïåðåìåùåíèÿ è îäíîâðåìåííî íîìåð îòáðîøåííîé "ëèøíåé" ñâÿçè, âòîðîé óêàçûâàåò íà ïðè÷èíó ïåðåìåùåíèÿ, òîãäà ïðè n ëèøíèõ ñâÿçåé ïîëó÷èì ñëåäóþùóþ ñèñòåìó óðàâíåíèé:

$$\left.\begin{array}{l} X_1 d_{11} + X_2 d_{12} + Õ_3 d_{13} + \ldots + X_n d_{1n} + D_{1P} = 0; \\ X_1 d_{21} + X_2 d_{22} + Õ_3 d_{23} + \ldots + X_n d_{2n} + D_{2P} = 0; \\ X_1 d_{31} + X_2 d_{32} + Õ_3 d_{33} + \ldots + X_n d_{3n} + D_{3P} = 0; \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots; \\ X_1 d_{n1} + X_2 d_{n2} + Õ_3 d_{n3} + \ldots + X_n d_{nn} + D_{nP} = 0. \end{array}\right\} \quad (1)$$

Ôîðìóëà (8.1) ïðåäñòàâëÿåò ñîáîé *êàíîíè÷åñêèå óðàâíåíèÿ ìåòîäà ñèë.* Ýòî íàçâàíèå óêàçûâàåò íà òî, ÷òî ýòè óðàâíåíèÿ ïèøóòñÿ ïî îïðåäåëåííîìó ïðàâèëó (êàíîíó) è ÷òî íåèçâåñòíûìè â ýòèõ óðàâíåíèÿõ ÿâëÿþòñÿ ñèëû, ïðåäñòàâëÿþùèå ñîáîé ðåàêöèè îòáðîøåííûõ ñâÿçåé. *×èñëî óðàâíåíèé äîëæíî áûòü ðàâíî ÷èñëó îòáðîøåííûõ ñâÿçåé, òî åñòü ðàâíî ñòåïåíè ñòàòè÷åñêîé íåîïðåäåëèìîñòè çàäàííîé ñèñòåìû.* Òàê, ïðè ñòåïåíè ñòàòè÷åñêîé íåîïðåäåëèìîñòè n = 1 ôîðìóëà (1) ïðèìåò âèä:

$$X_1 d_{11} + D_{1P} = 0. \quad (2)$$

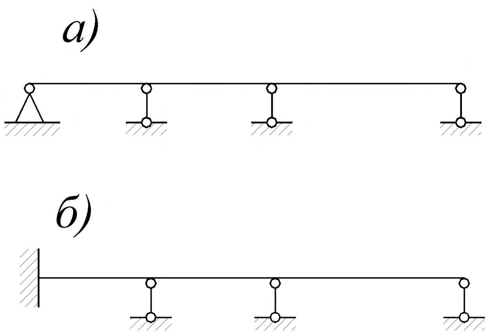
$$\left.\begin{array}{l} X_1 d_{11} + X_2 d_{12} + D_{1P} = 0 \\ X_1 d_{21} + X_2 d_{22} + D_{2P} = 0 \end{array}\right\} \quad (3)$$

Äëÿ ñèñòåìû ñî ñòåïåíüþ ñòàòè÷åñêîé íåîïðåäåëèìîñòè n = 2 áóäåì èìåòü çàâèñèìîñòü (3).

2. Íåðàçðåçíûå áàëêè

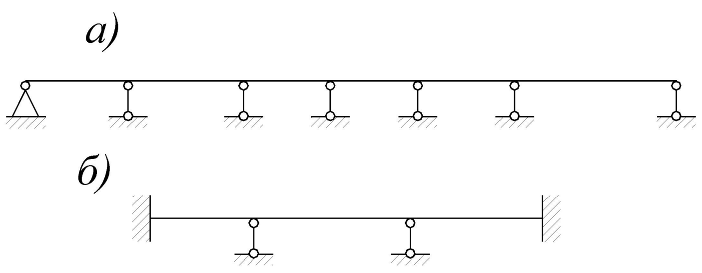
Äëÿ óìåíüøåíèÿ íàïðÿæåíèé â áàëêàõ ÷àñòî óâåëè÷èâàþò êîëè÷åñòâî îïîð, è îíè ñòàíîâÿòñÿ òîãäà ñòàòè÷åñêè íåîïðåäåëèìûìè. Ïîäîáíûå áàëêè äîâîëüíî ÷àñòî ïðèìåíÿþòñÿ â êà÷åñòâå ýëåìåíòîâ êîíñòðóêöèé ìàøèí è ðàçëè÷íûõ ñîîðóæåíèé.

Ñòåïåíü ñòàòè÷åñêîé íåîïðåäåëèìîñòè áàëîê â ñâÿçè ñ òåì, ÷òî â ðåàëüíûõ êîíñòðóêöèÿõ ÷àùå âñåãî âíåøíèå íàãðóçêè íàïðàâëåíû ïåðïåíäèêóëÿðíî ê èõ îñÿì, îïðåäåëÿåòñÿ ïî ÷èñëó "ëèøíèõ" îïîð. Ñòåïåíü ñòàòè÷åñêîé íåîïðåäåëèìîñòè áàëêè n=2 (Ðèñ. 4, à). Ñòåïåíü ñòàòè÷åñêîé íåîïðåäåëèìîñòè áàëêè n = 3 (Ðèñ. 4, á).



Ðèñ. 4

Íàèáîëåå ðàñïðîñòðàíåííûì òèïîì ñòàòè÷åñêè íåîïðåäåëèìûõ êîíñòðóêöèé ÿâëÿþòñÿ íåðàçðåçíûå áàëêè. *Íåðàçðåçíîé íàçûâàþò áàëêó, ïðîõîäÿùóþ íå ïðåðûâàÿñü, íàä ðÿäîì ïðîìåæóòî÷íûõ îïîð, ñ êîòîðûìè îíè ñîåäèíåíà øàðíèðíî.* Êðàéíèå îïîðû ïðè ýòîì ìîãóò áûòü èëè øàðíèðíûìè (Ðèñ. 5, à), èëè çàùåìëåííûìè (Ðèñ. 5, á).

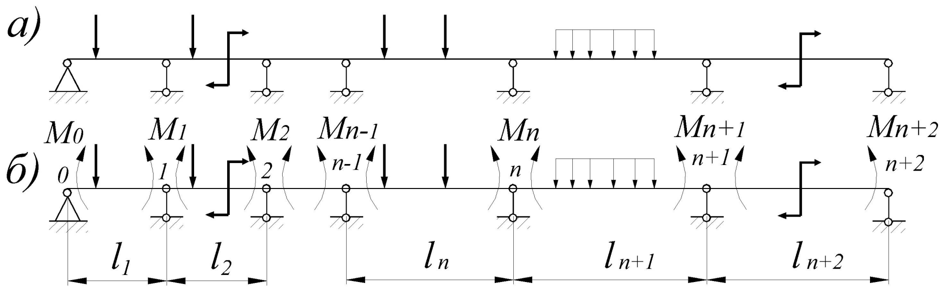


Ðèñ. 5

Îñíîâíóþ ñèñòåìó äëÿ íåðàçðåçíûõ áàëîê ïðèíèìàþò ïóòåì âðåçàíèÿ äîïîëíèòåëüíîãî øàðíèðà â îïîðó. Òîãäà ëèøíèìè íåèçâåñòíûìè áóäóò îïîðíûå èçãèáàþùèå ìîìåíòû íàä âñåìè ïðîìåæóòî÷íûìè îïîðàìè. Èõ îïðåäåëÿþò ñ ïîìîùüþ *óðàâíåíèÿ òðåõ ìîìåíòîâ.*

3. Óðàâíåíèå òðåõ ìîìåíòîâ

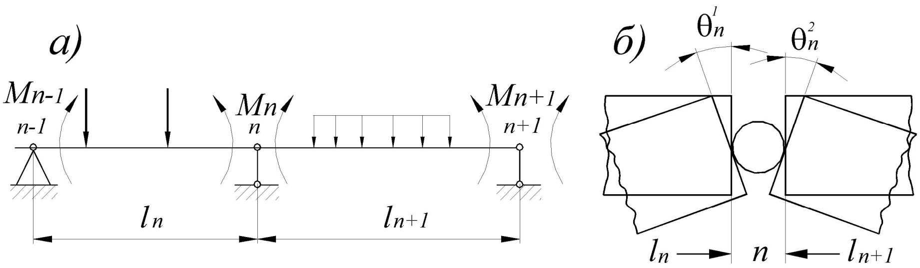
Äëÿ âûâîäà óðàâíåíèÿ òðåõ ìîìåíòîâ âîçüìåì íåðàçðåçíóþ áàëêó ñ ðÿäîì ïðîëåòîâ ðàçëè÷íîé äëèíû, íàãðóæåííóþ âåðòèêàëüíûìè ñèëàìè (Ðèñ. 6, à). Ïðèìåì îñíîâíóþ è ýêâèâàëåíòíóþ ñèñòåìó.



Ðèñ. 6

Íóìåðàöèþ îïîð è ïðîëåòîâ áóäåì âåñòè ñëåâà íàïðàâî, îáîçíà÷àÿ êðàéíþþ ëåâóþ îïîðó íîìåðîì "0", à äëèíó êðàéíåãî ëåâîãî ïðîëåòà íîìåðîì "1" (Ðèñ. 6, á). Ñå÷åíèÿ áàëêè âî âñåõ ïðîëåòàõ áóäåì ñ÷èòàòü îäèíàêîâûìè è, ñëåäîâàòåëüíî, æåñòêîñòü EI ÿâëÿåòñÿ ïîñòîÿííîé.

Ñîñòàâèì äåôîðìàöèîííîå óðàâíåíèå, âíîñÿùåå òå æå îãðàíè÷åíèÿ íà äåôîðìàöèè îñíîâíîé ñèñòåìû, êîòîðûå èìåþòñÿ â íåðàçðåçíîé áàëêå. Â îñíîâíîé ñèñòåìå îáå ñòîðîíû n–ãî îïîðíîãî ñå÷åíèÿ, ðàçäåëåííûå ïîñòàâëåííûì â áàëêó øàðíèðîì, ìîãóò ïîâîðà÷èâàòüñÿ ïîä íàãðóçêîé íåçàâèñèìî äðóã îò äðóãà (Ðèñ. 7., à).



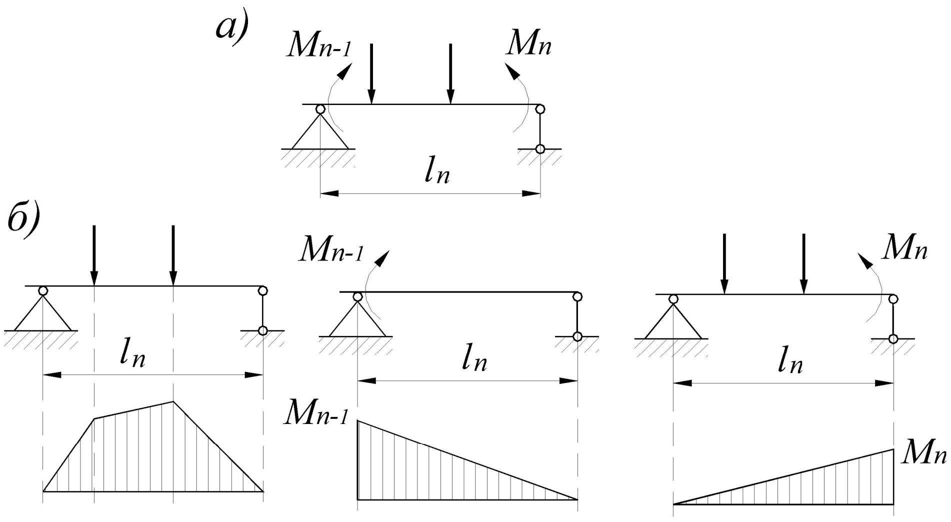
Ðèñ. 7

Óãîë ïîâîðîòà ñå÷åíèÿ íà îïîðå n ëåâîãî ïðèìûêàþùåãî ê äàííîé îïîðå ïðîëåòà îáîçíà÷èì q_n^1, à óãîë ïîâîðîòà ñå÷åíèÿ íà îïîðå n äëÿ ïðàâîãî ïðèìûêàþùåãî ïðîëåòà íàçîâåì q_n^2 (Ðèñ. 7, á).

Â íåðàçðåçíîé áàëêå îáà îïîðíûõ ñå÷åíèÿ ñîâïàäàþò è ïðåäñòàâëÿþò ñîáîé ëèøü ðàçíûå ñòîðîíû îäíîãî è òîãî æå îïîðíîãî ñå÷åíèÿ. Ñëåäîâàòåëüíî, óñëîâèåì ñîâìåñòíîñòè äåôîðìàöèé áóäåò:

$$q_n^1 = q_n^2.$$

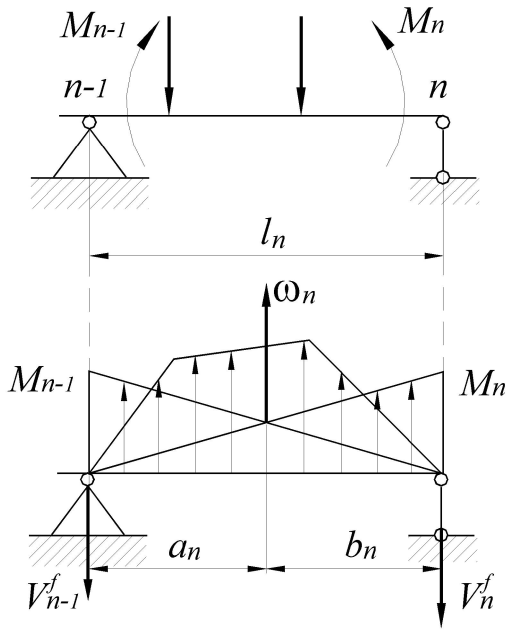
Óãëû ïîâîðîòà ñå÷åíèé â îñíîâíîé ñèñòåìå íà îïîðå n çàâèñÿò îò äåôîðìàöèé äâóõ ñìåæíûõ ïðîëåòîâ äëèíîé l_n è l_{n+1}. Ðàññìîòðèì ýòè äâà ïðîëåòà â îòäåëüíîñòè ñî âñåìè äåéñòâóþùèìè íà íèõ íàãðóçêàìè (Ðèñ. 8, à; Ðèñ. 10, à). Äëÿ âû÷èñëåíèÿ óãëîâ q_n^1 è q_n^2 âîñïîëüçóåìñÿ ãðàôîàíàëèòè÷åñêèì ìåòîäîì.



Ðèñ. 8

Ñòðîèì ýïþðû èçãèáàþùèõ ìîìåíòîâ äëÿ ïðîëåòà äëèíîé l_n â îòäåëüíîñòè îò âíåøíèõ íàãðóçîê è îïîðíûõ èçãèáàþùèõ ìîìåíòîâ M_{n-1} è Ì$_n$ (Ðèñ. 8, á).

Äëÿ ïðîëåòà äëèíîé l_n ïðèíèìàåì ôèêòèâíóþ áàëêó è íàãðóæàåì åå ýïþðàìè èçãèáàþùèõ ìîìåíòîâ (Ðèñ. 9).



Ðèñ. 9

Ôèêòèâíûìè áóäóò ñëåäóþùèå íàãðóçêè:

–ýïþðà èçãèáàþùèõ ìîìåíòîâ îò âíåøíèõ ñèë ïëîùàäüþ w_n ñ ðàññòîÿíèåì öåíòðà òÿæåñòè ýòîé ïëîùàäè îò ëåâîé îïîðû a_n;

–òðåóãîëüíàÿ ýïþðà èçãèáàþùèõ ìîìåíòîâ îò ïîëîæèòåëüíîãî îïîðíîãî ìîìåíòà M_{n-1} ñ ðàññòîÿíèåì öåíòðà òÿæåñòè ýòîé ïëîùàäè îò ëåâîé îïîðû $(1/3)l_n$;

–òðåóãîëüíàÿ ýïþðà èçãèáàþùèõ ìîìåíòîâ îò ïîëîæèòåëüíîãî îïîðíîãî ìîìåíòà Ì$_n$ ñ ðàññòîÿíèåì öåíòðà òÿæåñòè ýòîé ïëîùàäè îò ëåâîé îïîðû $(2/3)l_n$.

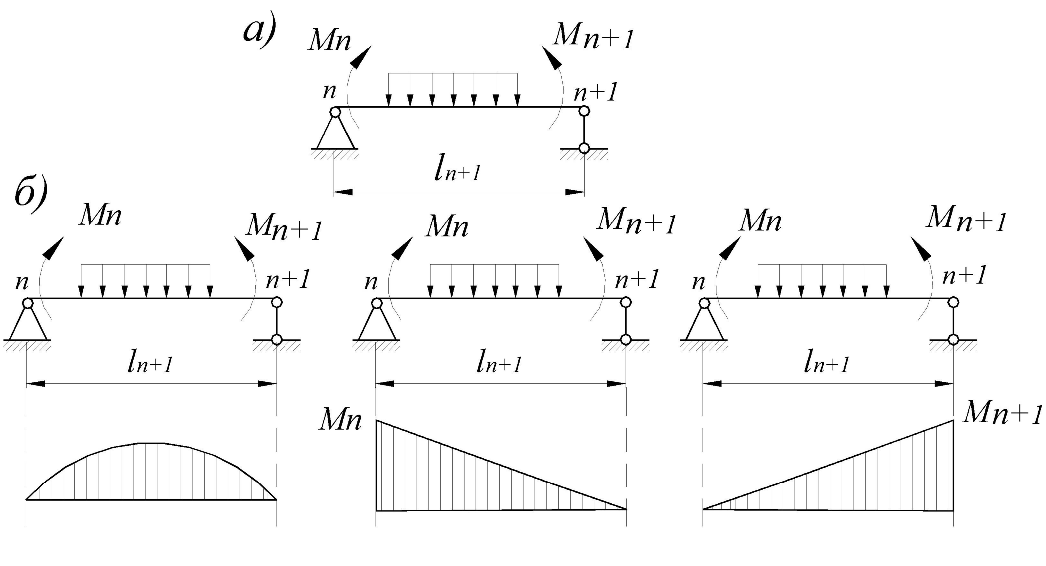
Òàê êàê $q_n^1 = Q_f^n/EI$ [ñì. ôîðìóëó (7.8)], a $Q_f^n = V_n^f$, îïðåäåëÿåì V_n^f. Ñîñòàâèì äëÿ ýòîãî óðàâíåíèå ðàâíîâåñèÿ $SM_{n-1} = 0$:

$SM_{n-1} = w_n a_n + (1/2)M_n l_n (2/3) l_n + (1/2)M_{n-1} l_n (2/3) l_n - V_n^f\ l_n = 0$, îòñþäà

$$V_n^f = w_n a_n / l_n + (1/3)M_n l_n + (1/6)M_{n-1} l_n, \text{ òîãäà}$$

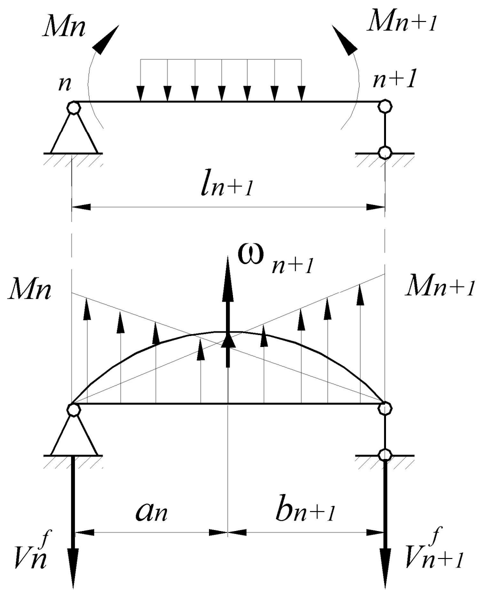
$$q_n^1 = Q_f^n/EI = V_n^f/EI = (1/EI)[(w_n a_n / l_n) + (1/3)M_n l_n + (1/6)M_{n-1} l_n].$$

Ñòðîèì ýïþðû èçãèáàþùèõ ìîìåíòîâ äëÿ ïðîëåòà äëèíîé l_{n+1} â îòäåëüíîñòè îò âíåøíèõ íàãðóçîê è îïîðíûõ èçãèáàþùèõ ìîìåíòîâ Ì$_n$ è M_{n+1} (Ðèñ. 10, á).



Ðèñ. 10

Ïðèíèìàåì ôèêòèâíóþ áàëêó è íàãðóæàåì åå ýïþðàìè èçãèáàþùèõ ìîìåíòîâ (Ðèñ. 11).



Đèñ. 11

Ôèêòèâíûìè áóäóò ñëåäóþùèå íàãðóçêè:

–ýïþðà èçãèáàþùèõ ìîìåíòîâ îò âíåøíèõ ñèë ïëîùàäüþ w_{n+1} ñ ðàññòîÿíèåì öåíòðà òÿæåñòè ýòàï ïëîùàäè îò ïðèâîé îïîðû b_{n+1};

–òðåóãîëüíàÿ ýïþðà èçãèáàþùèõ ìîìåíòîâ îò ïîëîæèòåëüíîãî îïîðíîãî ìîìåíòà Ì_{n+1} ñ ðàññòîÿíèåì öåíòðà òÿæåñòè ýòîé ïëîùàäè îò ïðàâîé îïîðû $(1/3)l_{n+1}$;

–òðåóãîëüíàÿ ýïþðà èçãèáàþùèõ ìîìåíòîâ îò ïîëîæèòåëüíîãî îïîðíîãî ìîìåíòà Ì_n ñ ðàññòîÿíèåì öåíòðà òÿæåñòè ýòîé ïëîùàäè îò ïðàâîé îïîðû $(2/3)l_{n+1}$.

Òàê êàê $q_n^2 = Q_f^n/EI$ [ñì. ôîðìóëó (7.8)], a $Q_f^n = V_n^f$, îïðåäåëÿåì V_n^f. Ñîñòàâèì äëÿ ýòîãî óðàâíåíèå ðàâíîâåñèÿ $S\text{Ì}_{n+1} = 0$:

$$SM_{n-1} = - w_{n+1}b_{n+1}+(1/2)M_nl_{n+1}(2/3)l_{n+1}+(1/2)M_{n+1}l_{n+1}(2/3)l_{n+1}-V_n^fl_{n+1} = 0,$$

îòñþäà $V_n^f = (w_{n+1}b_{n+1})/l_{n+1} + (1/3)M_nl_{n+1} + (1/6)M_{n+1}l_{n+1}$, òîãäà

$$q_n^2 = Q_f^n/EI = -V_n^f/EI = -(1/EI)[(w_{n+1}b_{n+1}/l_{n+1}+ (1/3)M_nl_{n+1}+(1/6) M_{n+1}l_{n+1}].$$

Ïîäñòàâëÿåì íàéäåííûå çíà÷åíèÿ óãëîâ ïîâîðîòà â óðàâíåíèå ñîâìåñòíîñòè äåôîðìàöèé $q_n^1 = q_n^2$:

$$(1/EI)[w_na_n/l_n + (1/3)M_nl_n + (1/6)M_{n-1}l_n] =$$

$$= -(1/EI)[w_{n+1}b_{n+1}/l_{n+1} + (1/3)M_n l_{n+1} + (1/6)M_{n+1}l_{n+1}],$$ èëè

$$(1/3)M_n l_n + (1/6)M_{n-1}l_n + (1/3)M_n l_{n+1} + (1/6)M_{n+1}l_{n+1} =$$

$$= -(w_n a_n/l_n + w_{n+1}b_{n+1}/l_{n+1}).$$

Ïðèâîäÿ ê îáùåìó çíàìåíàòåëþ, ïîëó÷èì:

$$M_{n-1}l_n + 2M_n(l_n + l_{n+1}) + M_{n+1}l_{n+1} = -6(w_n a_n/l_n + w_{n+1}b_{n+1}/l_{n+1}). \quad (4)$$

Ôîðìóëó (4) íàçûâàþò *óðàâíåíèåì òðåõ ìîìåíòîâ*.

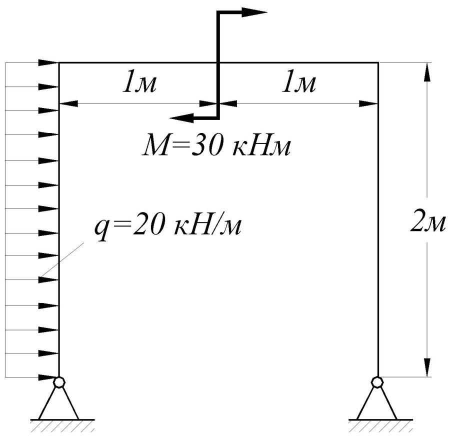
Òàê êàê èíäåêñû "n" è "n+1" îòíîñÿòñÿ ñîîòâåòñòâåííî ê ëåâîìó è ïðàâîìó ïðîëåòàì, ôîðìóëó (4) ìîæíî ïåðåïèñàòü â ñëåäóþùåì âèäå:

$$M_{\text{ëåâ}}l_{\text{ëåâ}} + 2M_{\text{ñð}}(l_{\text{ëåâ}} + l_{\text{ïð}}) + M_{\text{ïð}}l_{\text{ïð}} = -6(w_{\text{ëåâ}}a_{\text{ëåâ}}/l_{\text{ëåâ}} + w_{\text{ïð}}b_{\text{ïð}}/l_{\text{ïð}}). \quad (8.5)$$

Òàêèõ óðàâíåíèé ìû ìîæåì íàïèñàòü ñòîëüêî, ñêîëüêî èìååì íåèçâåñòíûõ îïîðíûõ ìîìåíòîâ. Ïîñëå âû÷èñëåíèÿ îïîðíûõ ìîìåíòîâ çàäà÷à ñâîäèòñÿ ê ðàñ÷åòó ðÿäà øàðíèðíî îïåðòûõ áàëîê, íàãðóæåííûõ óæå èçâåñòíûìè îïîðíûìè ìîìåíòàìè è âíåøíåé íàãðóçêîé.

Ïðèìåðû ðàñ÷åòà

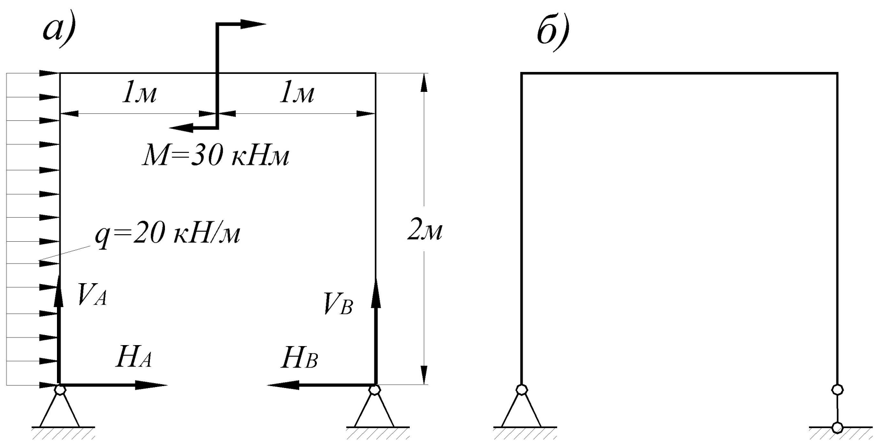
Ïðèìåð 1. Ðàñêðûòü ñòàòè÷åñêóþ íåîïðåäåëèìîñòü çàäàííîé ðàìû (). Ïåðåìåùåíèÿ, âõîäÿùèå â êàíîíè÷åñêèå óðàâíåíèå ìåòîäà ñèë îïðåäåëèòü ñ ïîìîùüþ ìåòîäà Âåðåùàãèíà.



Ðèñ. 12

Ð å ø å í è å. 1. Îïðåäåëÿåì ñòåïåíü ñòàòè÷åñêîé íåîïðåäåëèìîñòè, îáîçíà÷èâ íà ÷åðòåæå âîçìîæíûå îïîðíûå ðåàêöèè (Ðèñ. 13, à).

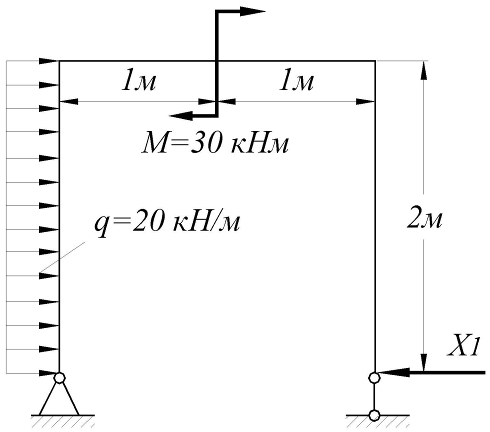
$$n = m – k = 4 – 3 = 1.$$



Ðèñ. 13

2. Ïðèíèìàåì îñíîâíóþ ñèñòåìó äëÿ ðàñ÷åòà, îòáðàñûâàÿ ãîðèçîíòàëüíóþ ñâÿçü íà ïðàâîì êîíöå ðàìû. Äëÿ ýòîãî çàìåíèì øàðíèðíî íåïîäâèæíóþ îïîðó øàðíèðíî ïîäâèæíîé (Ðèñ. 13, á).

3. Ïðèíèìàåì ýêâèâàëåíòíóþ ñèñòåìó (Ðèñ. 14). Ïðèêëàäûâàåì âìåñòî ðåàêöèè îòáðîøåííîé ñâÿçè íåèçâåñòíîå óñèëèå $Õ_1$, îãðàíè÷èâàþùåå ãîðèçîíòàëüíîå ïåðåìåùåíèå ïðàâîãî êîíöà ðàìû è âíåøíèå íàãðóçêè.



Ðèñ. 13

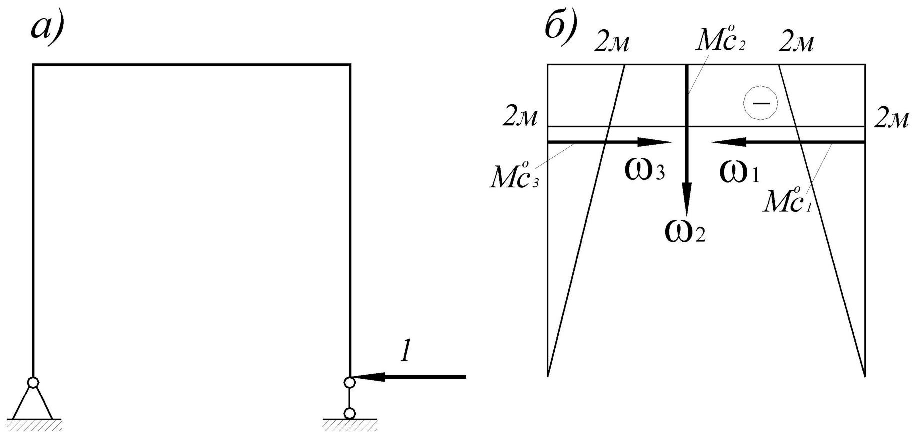
4. Çàïèñûâàåì êàíîíè÷åñêîå óðàâíåíèå ìåòîäà ñèë [ñì. ôîðìóëó (2)]:

$C_1 d_{11} + D_{1Ð} = 0$, â êîòîðîå âõîäÿò ñëåäóþùèå ïåðåìåùåíèÿ:

d_{11}– *ïåðåìåùåíèå ïðàâîãî êîíöà ðàìû â ãîðèçîíòàëüíîì íàïðàâëåíèè îò äåéñòâèÿ åäèíè÷íîé ñèëû;*

$D_{1Ð}$– *ïåðåìåùåíèå ïðàâîãî êîíöà ðàìû â ãîðèçîíòàëüíîì íàïðàâëåíèè îò äåéñòâèÿ âíåøíèõ ñèë.*

5. Îïðåäåëÿåì ïåðåìåùåíèå d_{11}. Ïðèêëàäûâàåì ê îñíîâíîé ñèñòåìå â ïðàâîé îïîðå ãîðèçîíòàëüíóþ åäèíè÷íóþ ñèëó â êà÷åñòâå âíåøíåé íàãðóçêè è ñòðîèì ýïþðó èçãèáàþùèõ ìîìåíòîâ (Ðèñ. 15, à). Îïðåäåëÿåì åå ïëîùàäè è îðäèíàòû $Ì_{Ñ}^{0}$, ïðîõîäÿùèå ÷åðåç èõ öåíòðû òÿæåñòè (Ðèñ. 15, á), òàê êàê ïîñëå ïîâòîðíîãî ïðèëîæåíèÿ åäèíè÷íîé ñèëû ìû ïîëó÷èì òî÷íî òàêóþ æå ýïþðó èçãèáàþùèõ ìîìåíòîâ.



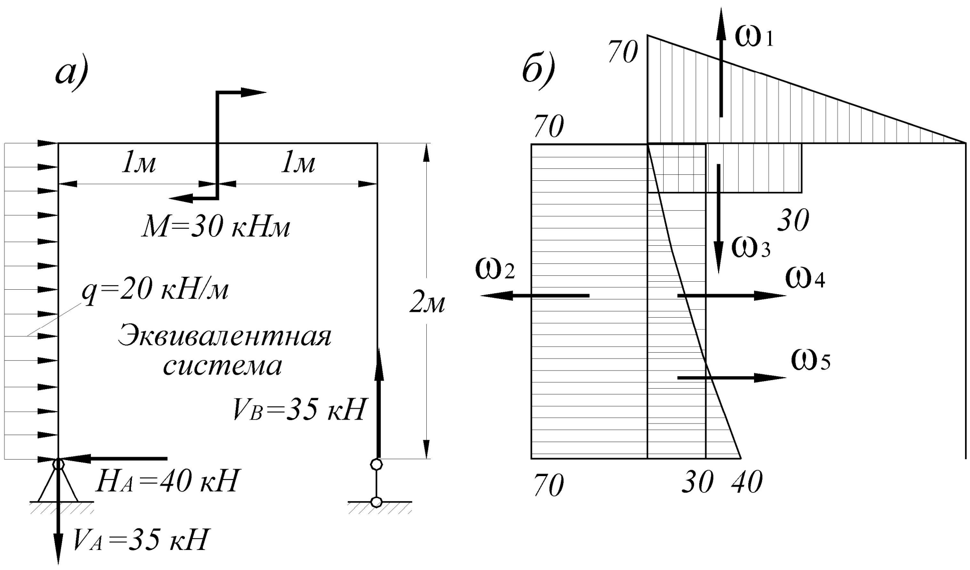
Ðèñ. 15

w_1 = w_3 = – (1/2)×2×2 = –2 ì^2; w_2 = –2×2 = – 4 ì^2;

$Ì_{Ñ1}^0$ = $Ì_{Ñ3}^0$ = – 4/3 ì; $Ì_{Ñ2}^0$ = –2 ì, òîãäà ïî ôîðìóëå (7.21):

d_{11} = (1/EI)($w_1 Ì_{Ñ1}^0$ + $w_2 Ì_{Ñ2}^0$ + $w_3 Ì_{Ñ3}^0$) = (1/EI)[2×2(4/3) + 4×2] = 40/EI ì/êÍ.

6. Îïðåäåëÿåì ïåðåìåùåíèå $D_{1Ð}$. Ïðèêëàäûâàåì ê îñíîâíîé ñèñòåìå âíåøíèå íàãðóçêè, îïðåäåëÿåì âåëè÷èíû îïîðíûõ ðåàêöèé (Ðèñ. 16, à), ñòðîèì ðàññëîåííóþ ýïþðó èçãèáàþùèõ ìîìåíòîâ è îïðåäåëÿåì ñîîòâåòñòâóþùèå åå ïëîùàäè (Ðèñ. 16, á).



Ðèñ. 16

w_1 = (1/2)×70×2 = 70 êÍì^2; w_2 = 70×2 = 140 êÍì^2; w_3 = –30×1 = –30 êÍì^2; w_4 = –30×2 = – 60 êÍì^2; w_5 = – (1/3)×40×2 = – 80/3 êÍì^2.

Ñòðîèì ýïþðó èçãèáàþùèõ ìîìåíòîâ îò åäèíè÷íîé ñèëû, ïðèëîæåííîé â ïðàâîé îïîðå ãîðèçîíòàëüíî (Ðèñ. 8.17, à), è îïðåäåëÿåì îðäèíàòû Ì$_{Ñ}^{0}$, ïîëó÷åííûå ïîñëå ïðîåöèðîâàíèÿ íà íåå öåíòðîâ ýïþðû îò âíåøíèõ ñèë (Ðèñ.8.17, á).

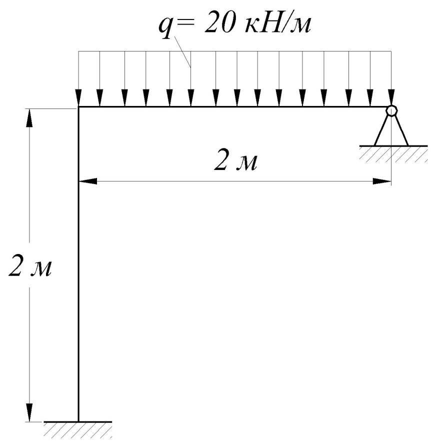
Ì$_{Ñ1}^{0}$ = –2 ì; Ì$_{Ñ2}^{0}$ = –1 ì; Ì$_{Ñ3}^{0}$ = –2 ì; Ì$_{Ñ4}^{0}$ = –1 ì; Ì$_{Ñ5}^{0}$ = –2(1/4) = –1/2, òîãäà D$_{1Ð}$ = (1/EI)(w$_1$Ì$_{Ñ1}^{0}$ + w$_2$Ì$_{Ñ2}^{0}$ + w$_3$Ì$_{Ñ3}^{0}$ + w$_4$Ì$_{Ñ4}^{0}$ + w$_5$Ì$_{Ñ5}^{0}$) = (1/EI)[–70×2 – – 140×1 + 30×2 + 60×1 + (80/3)(1/2)] = – (440/3EI) ì.

7.Ïîäñòàâëÿåì çíà÷åíèÿ íàéäåííûõ ïåðåìåùåíèé d$_{11}$ è D$_{1Ð}$ â êàíîíè÷åñêîå óðàâíåíèå ìåòîäà ñèë.

Õ$_1$(40/3EI) – (440/3EI) = 0, èëè 40Õ$_1$ = 440, òîãäà Õ$_1$ = 11 êÍ.

Ïðèìåð 2. Ðàñêðûòü ñòàòè÷åñêóþ íåîïðåäåëèìîñòü çàäàííîé ðàìû (Ðèñ. 18) è ïîñòðîèòü ýïþðû ïîïåðå÷íûõ ñèë, èçãèáàþùèõ ìîìåíòîâ è íîðìàëüíûõ ñèë. Ïðîèçâåñòè ïðîâåðêó ïðàâèëüíîñòè îïðåäåëåíèÿ ðåàêöèé ëèøíèõ ñâÿçåé.

Ïåðåìåùåíèÿ, âõîäÿùèå â êàíîíè÷åñêèå óðàâíåíèÿ ìåòîäà ñèë îïðåäåëèòü ñ ïîìîùüþ ìåòîäà Âåðåùàãèíà.

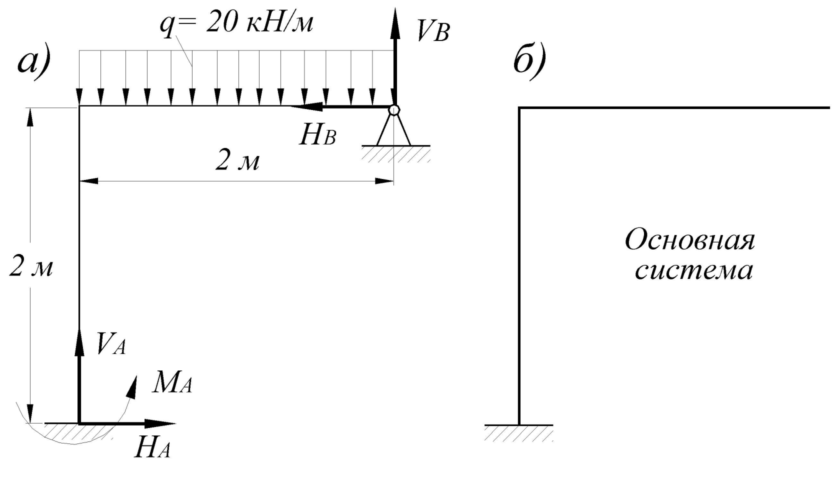


Ðèñ 18

Ð å ø å í è å. 1. Îïðåäåëÿåì ñòåïåíü ñòàòè÷åñêîé íåîïðåäåëèìîñòè, îáîçíà÷èâ íà ÷åðòåæå âîçìîæíûå îïîðíûå ðåàêöèè (Ðèñ. 19, à).

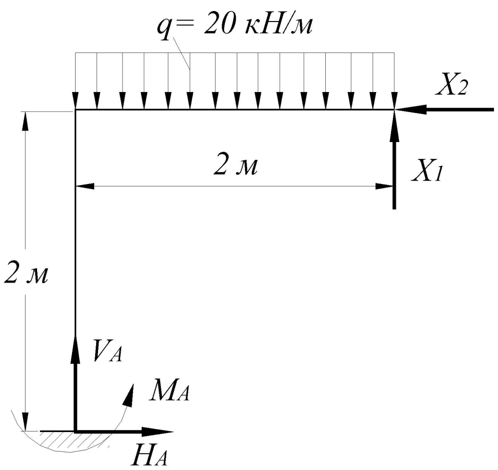
$$n = m - k = 5 - 3 = 2.$$

2. Ïðèíèìàåì îñíîâíóþ ñèñòåìó äëÿ ðàñ÷åòà, îòáðàñûâàÿ âåðòèêàëüíóþ è ãîðèçîíòàëüíóþ ñâÿçè íà ïðàâîì êîíöå ðàìû. Äëÿ ýòîãî âìåñòî øàðíèðíî íåïîäâèæíîé îïîðû îñòàâèì ñâîáîäíûé êîíåö (Ðèñ. 19, á).



Ðèñ 19

3. Ïðèíèìàåì ýêâèâàëåíòíóþ ñèñòåìó. Ïðèêëàäûâàåì âìåñòî ðåàêöèè îòáðîøåííûõ ñâÿçåé íåèçâåñòíûå óñèëèÿ Õ1 è Õ2, îãðàíè÷èâàþùèå âåðòèêàëüíîå è ãîðèçîíòàëüíîå ïåðåìåùåíèå ïðàâîãî êîíöà ðàìû, è âíåøíèå íàãðóçêè (Ðèñ. 20).



Ðèñ 20

4. Çàïèñûâàåì êàíîíè÷åñêîå óðàâíåíèå ìåòîäà ñèë [ñì. ôîðìóëó (3)]:

$C_1d_{11} + C_1d_{12} + D_{1Ð} = 0$;

$C_1d_{21} + C_2d_{22} + D_{2Ð} = 0$,

ãäå d_{11}– âåðòèêàëüíîå ïåðåìåùåíèå ïðàâîãî êîíöà ðàìû îò äåéñòâèÿ åäèíè÷íîé ñèëû, ïðèëîæåííîé â âåðòèêàëüíîì íàïðàâëåíèè;

d_{12}– âåðòèêàëüíîå ïåðåìåùåíèå ïðàâîãî êîíöà ðàìû îò äåéñòâèÿ åäèíè÷íîé ñèëû, ïðèëîæåííîé â ãîðèçîíòàëüíîì íàïðàâëåíèè;

d_{21}– ãîðèçîíòàëüíîå ïåðåìåùåíèå ïðàâîãî êîíöà ðàìû îò äåéñòâèÿ åäèíè÷íîé ñèëû, ïðèëîæåííîé â âåðòèêàëüíîì íàïðàâëåíèè;

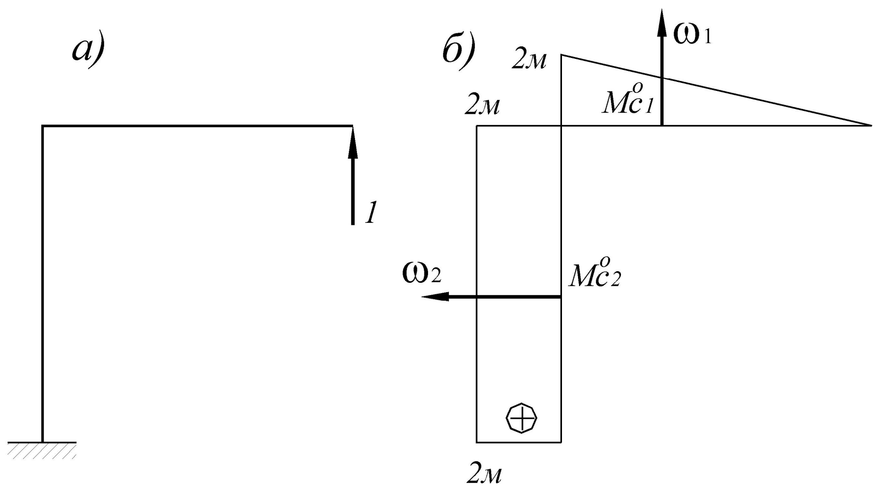
d_{22}– ãîðèçîíòàëüíîå ïåðåìåùåíèå ïðàâîãî êîíöà ðàìû îò äåéñòâèÿ åäèíè÷íîé ñèëû, ïðèëîæåííîé â ãîðèçîíòàëüíîì íàïðàâëåíèè;

$D_{1Ð}$– âåðòèêàëüíîå ïåðåìåùåíèå ïðàâîãî êîíöà ðàìû îò äåéñòâèÿ âíåøíèõ ñèë;

$D_{2Ð}$– ãîðèçîíòàëüíîå ïåðåìåùåíèå ïðàâîãî êîíöà ðàìû îò äåéñòâèÿ âíåøíèõ ñèë.

5. Îïðåäåëÿåì ïåðåìåùåíèå d_{11}. Ïðèêëàäûâàåì ê îñíîâíîé ñèñòåìå ê ïðàâîìó êîíöó âåðòèêàëüíóþ åäèíè÷íóþ ñèëó â êà÷åñòâå âíåøíåé íàãðóçêè

(Ðèñ. 21, à) è ñòðîèì ýïþðó èçãèáàþùèõ ìîìåíòîâ. Îïðåäåëÿåì åå ïëîùàäè è îðäèíàòû $Ì_Ñ^0$, ïðîõîäÿùèå ÷åðåç èõ öåíòðû òÿæåñòè (Ðèñ. 21, á), òàê êàê ïîñëå ïîâîðîòà ïðèëîæåíèÿ åäèíè÷íîé ñèëû ìû ïîëó÷èì òî÷íî òàêóþ æå ýïþðó èçãèáàþùèõ ìîìåíòîâ.

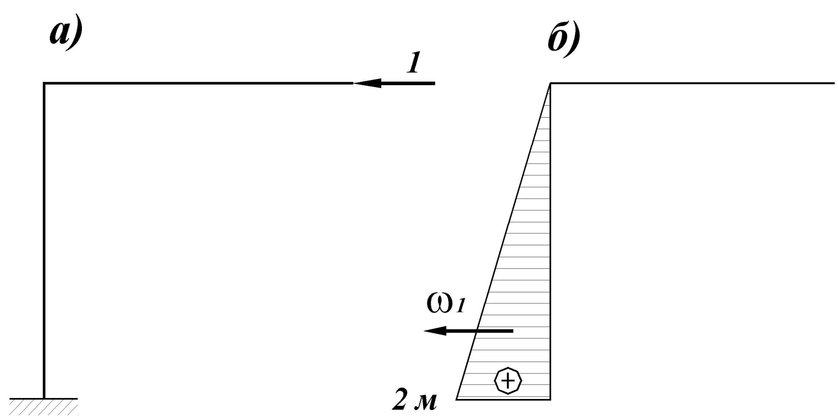


Ðèñ 21

$w_1 = (1/2)\times 2\times 2 = 2$ ì^2; $Ì_{Ñ1}^0 = 4/3$ ì; $w_2 = 2\times 2 = 4$ ì^2; $Ì_{Ñ2}^0 = 2$ ì, òîãäà

$d_{11} = (1/EI)[2\times(4/3) + 4\times 2] = 32/3EI$ ì/êÍ.

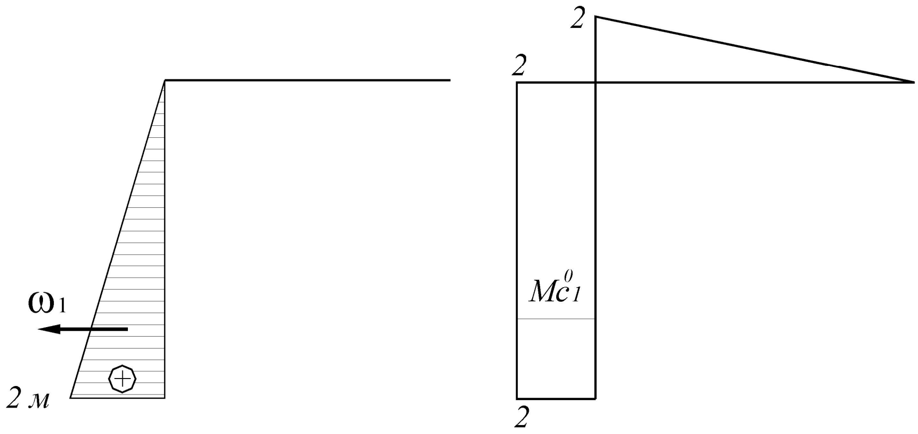
6. Îïðåäåëÿåì ïåðåìåùåíèå d_{12}. Ïðèêëàäûâàåì ê ïðàâîìó êîíöó ðàìû ãîðèçîíòàëüíóþ åäèíè÷íóþ ñèëó (Ðèñ. 22, à), ñòðîèì ýïþðó èçãèáàþùèõ ìîìåíòîâ è îïðåäåëÿåì åå ïëîùàäü (Ðèñ. 22, á).



Ðèñ 22

$w_1 = (1/2)\times 2\times 2 = 2$ ì^2.

Îïðåäåëÿåì îðäèíàòó Ì$_{Ñ1}^{0}$, ïîëó÷åííóþ ïðè ïðîåöèðîâàíèè öåíòðà òÿæåñòè ïëîùàäè w$_1$ (Ðèñ. 23, à) íà ýïþðó, ïîñòðîåííóþ îò äåéñòâèÿ âåðòèêàëüíîé åäèíè÷íîé ñèëû (Ðèñ. 23, á).

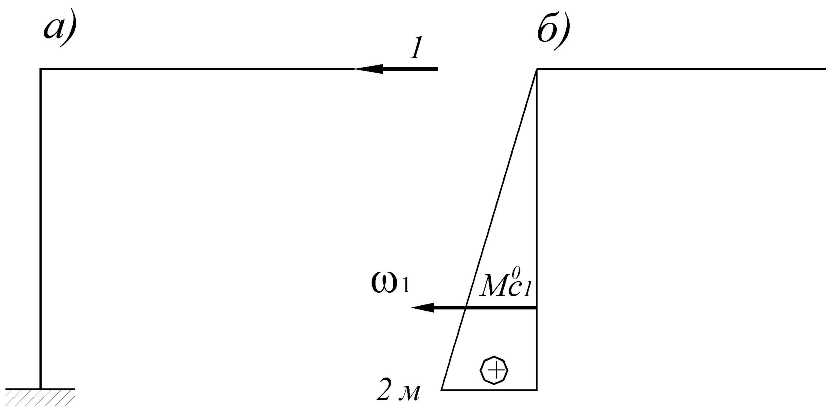


Ðèñ. 23

Ì$_{Ñ1}^{0}$ = 2 ì, òîãäà d$_{11}$ = (1/EI)2×2 = 4/EI ì/êÍ.

Ïåðåìåùåíèå d$_{21}$ áóäåò ðàâíî ïåðåìåùåíèþ d$_{12}$, òàê êàê îáðàòíîå ïåðåìíîæåíèå ýïþð äàñò òàêîé æå ðåçóëüòàò, òî åñòü d$_{21}$ = 4/EI ì/êÍ.

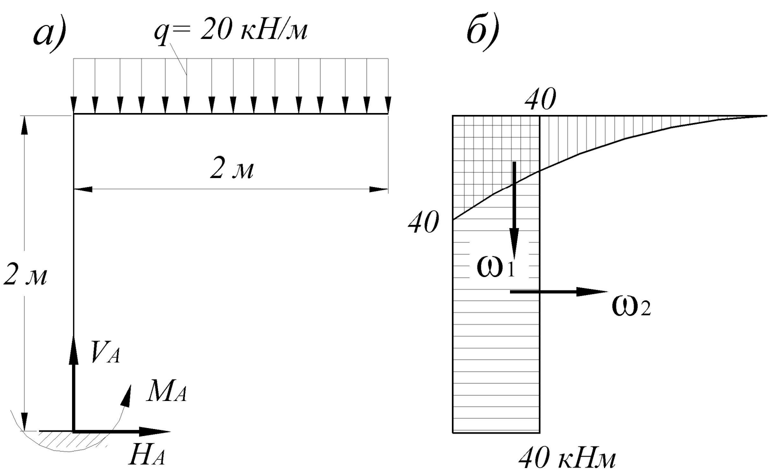
7. Îïðåäåëÿåì ïåðåìåùåíèå d$_{22}$. Ïðèêëàäûâàåì ê îñíîâíîé ñèñòåìå ê ïðàâîìó êîíöó ãîðèçîíòàëüíóþ åäèíè÷íóþ ñèëó â êà÷åñòâå âíåøíåé íàãðóçêè (Ðèñ. 24, à) è ñòðîèì ýïþðó èçãèáàþùèõ ìîìåíòîâ. Îïðåäåëÿåì åå ïëîùàäü è îðäèíàòó Ì$_{Ñ}^{0}$, ïðîõîäÿùóþ ÷åðåç åå öåíòð òÿæåñòè (Ðèñ. 24, á).



Ðèñ 24

w_1 = 2 ì^2; Ì$_{Ñ1}^0$ = 4/3, òîãäà d_{22} = (1/EI)×[2×(4/3)] = 8/3EI ì/êÍ.

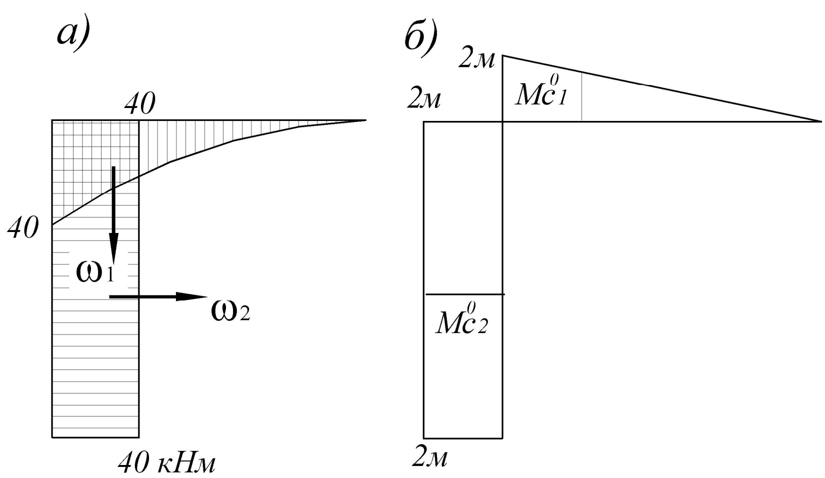
8. Îïðåäåëÿåì ïåðåìåùåíèå $D_{1Ð}$. Ïðèêëàäûâàåì ê îñíîâíîé ñèñòåìå âíåøíþþ íàãðóçêó (Ðèñ. 25, à), ñòðîèì ýïþðó èçãèáàþùèõ ìîìåíòîâ è îïðåäåëÿåì åå ïëîùàäè (Ðèñ. 25, á).



Ðèñ. 25

w_1 = (1/3)×40×2 = – 80/3 êÍì^2; w_2 = – 40×2 = – 80 êÍì^2.

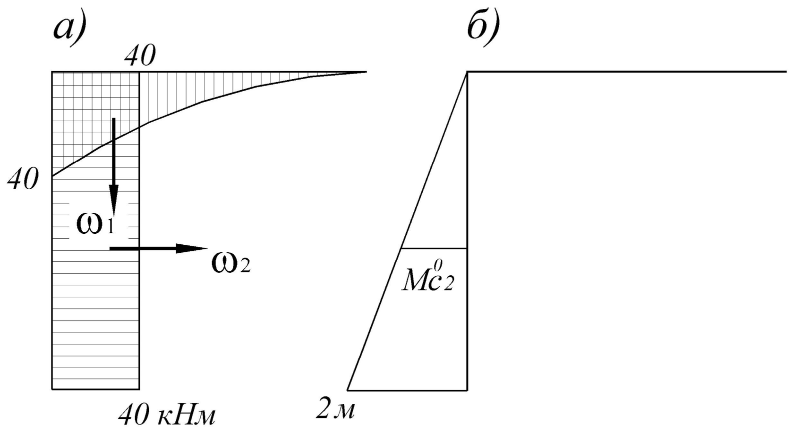
Ñòðîèì ýïþðó èçãèáàþùèõ ìîìåíòîâ îò åäèíè÷íîé ñèëû, ïðèëîæåííîé ê ïðàâîìó êîíöó ðàìû âåðòèêàëüíî (Ðèñ. 26, à), è îïðåäåëÿåì îðäèíàòû Ì$_Ñ^0$, ïîëó÷åííûå ïîñëå ïðîâåäåíèÿ íà íåå öåíòðîâ òÿæåñòè ïëîùàäåé ýïþðû îò âíåøíèõ ñèë (Ðèñ. 26, á).



Ðèñ. 26

Ì$_{Ñ1}^{0}$ = 3/2 ì; Ì$_{Ñ2}^{0}$ = 2 ì, òîãäà D$_{1Ð}$ = (1/EI)[–(80/3)(3/2) – 80×2] = –200/EI ì.

9. Îïðåäåëÿåì ïåðåìåùåíèå D$_{2Ð}$. Ñòðîèì ýïþðó èçãèáàþùèõ ìîìåíòîâ îò åäèíè÷íîé ñèëû (Ðèñ. 27, à), ïðèëîæåííîé ê ïðàâîìó êîíöó ðàìû ãîðèçîíòàëüíî è îïðåäåëÿåì îðäèíàòó Ì$_{Ñ2}^{0}$, ïîëó÷åííóþ ïîñëå ïðîåöèðîâàíèÿ íå íåå öåíòðà òÿæåñòè ïëîùàäè w$_2$ ýïþðû îò âíåøíèõ ñèë (Ðèñ. 27, á).



Ðèñ. 27

Ì$_{Ñ2}^{0}$ = 1 ì, òîãäà D$_{2Ð}$ = (1/EI) (– 80×1) = –80 / EI ì.

10. Ïîäñòàâëÿåì íàéäåííûå ïåðåìåùåíèÿ â êàíîíè÷åñêèå óðàâíåíèÿ ìåòîäà ñèë:

Õ$_1$(32/3EI) + Õ$_2$(4/EI) – (200/EI) = 0; Õ$_1$(4/EI) + Õ$_2$(8/3EI) – (80/EI) = 0, èëè

32Õ$_1$ + 12Õ$_2$ – 600 = 0½– 2
12Õ$_1$ + 8Õ$_2$ – 240 = 0½ 3

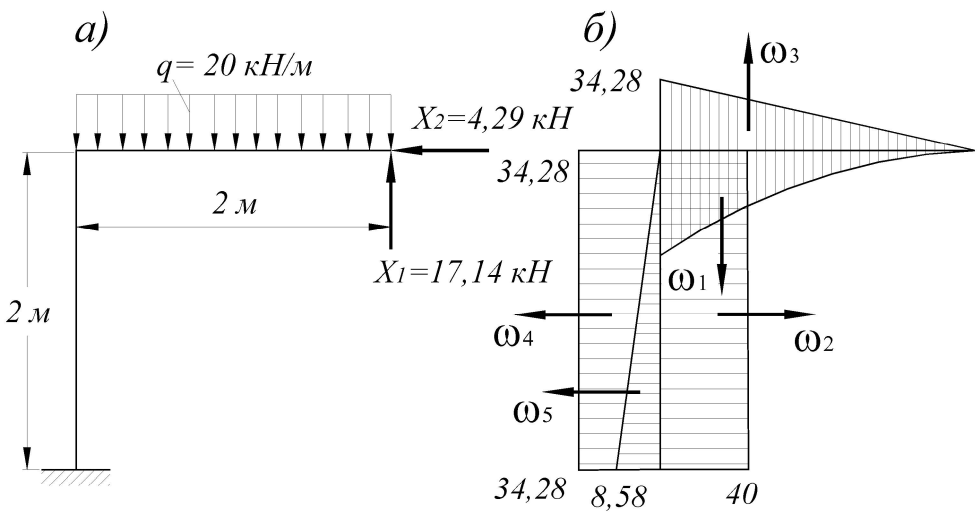
– 28Õ$_1$ + 480 = 0, ñëåäîâàòåëüíî, Õ$_1$ = 17,14 êÍ.

32Õ$_1$ + 12Õ$_2$ – 600 = 0½– 3
12Õ$_1$ + 8Õ$_2$ – 240 = 0½ 8

28Õ$_2$ – 120 = 0, ñëåäîâàòåëüíî, Õ» = 4,29 êÍ.

11. Ïðîâîäèì ïðîâåðêó ïðàâèëüíîñòè îïðåäåëåíèÿ ðåàêöèè îòáðîøåííûõ ñâÿçåé. Îïðåäåëèì ãîðèçîíòàëüíîå ïåðåìåùåíèå ïðàâîãî êîíöà ðàìû ñ ó÷åòîì èõ çíà÷åíèé (Ðèñ. 28, à). Î÷åâèäíî, ÷òî â ýòîì ñëó÷àå

ãîðèçîíòàëüíîå ïåðåìåùåíèå ïðàâîãî êîíöà ðàìû $õ_{Â}$ äîëæíî áûòü ðàâíî íóëþ. Ñòðîèì ðàññëîâåííóþ ýïþðó ñ ó÷åòîì âñåõ âíåøíèõ íàãðóçîê è îïðåäåëÿåì åå ïëîùàäè (Ðèñ. 28, á).

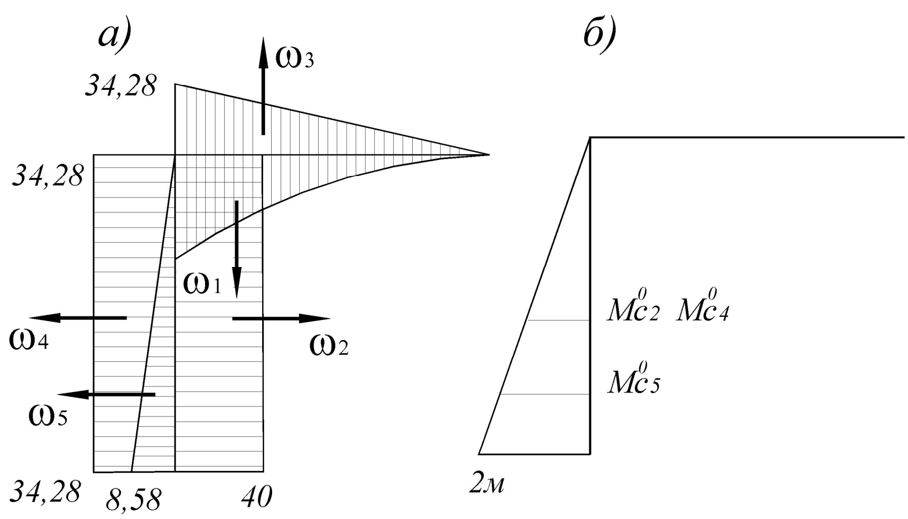


Ðèñ. 28

w_1 = –(1/3)×40×2= –80/3 êÍì^2; w_2= –40×2= –80 êÍì^2; w_3=(1/2)·34,28×2=34,28 êÍì^2;

w_4 = 34,28×2 = 68,56 êÍì^2; w_5 = (1/2)×8,58×2 = 8,58 êÍì^2.

Îïðåäåëÿåì âåëè÷èíó îðäèíàò $Ì_{Ñ}^{0}$ (Ðèñ. 29, á) íà ýïþðå èçãèáàþùèõ ìîìåíòîâ îò åäèíè÷íîé ñèëû, ïðèëîæåííîé ê ïðàâîìó êîíöó â ãîðèçîíòàëüíîì íàïðàâëåíèè (Ðèñ. 29, à).

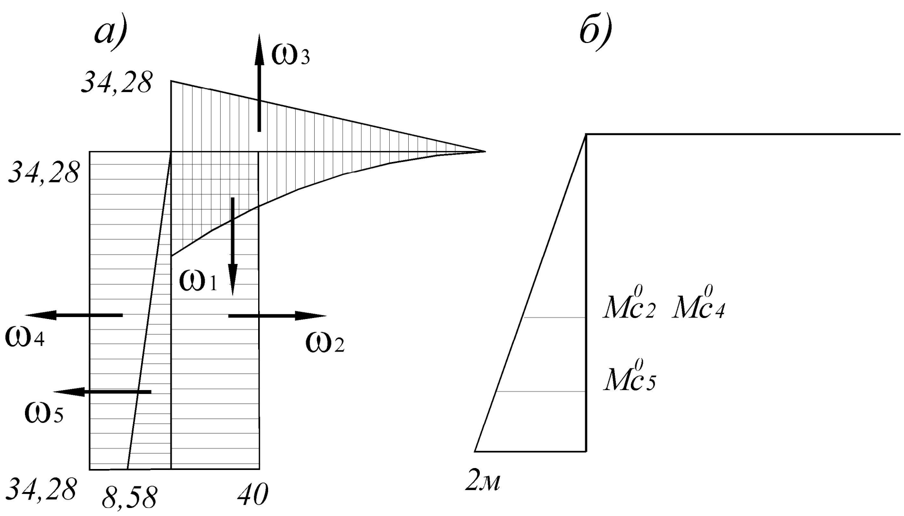


Ðèñ. 29

$Ì_{Ñ2}^{0}$ = $Ì_{Ñ4}^{0}$ = 1ì; $Ì_{Ñ5}^{0}$ = (4/3) ì, òîãäà

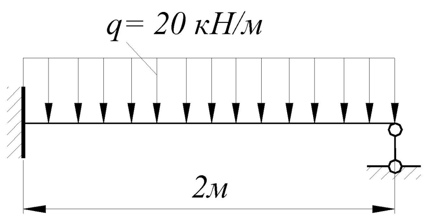
$õ_{Â}$ = (1/EI)($w_1 Ì_{Ñ1}^{0}$ + $w_2 Ì_{Ñ2}^{0}$ + $w_3 Ì_{Ñ3}^{0}$ + $w_4 Ì_{Ñ4}^{0}$ + $w_5 Ì_{Ñ5}^{0}$) = (1/EI) [– 80×1 + + 68,56×1 + 8,58 (4/3)] = 0.

12. Äëÿ çàäàííîé ðàìû (Ðèñ. 30, à) ñòðîèì ýïþðû ïîïåðå÷íûõ (Ðèñ.30,á), íîðìàëüíûõ ñèë (Ðèñ. 30, â) è èçãèáàþùèõ ìîìåíòîâ (Ðèñ.30,ã).



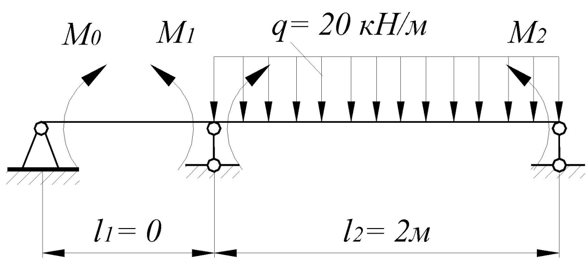
Ðèñ. 30

Ïðèìåð 3. Ðàñêðûòü ñòàòè÷åñêóþ íåîïðåäåëèìîñòü çàäàííîé áàëêè (Ðèñ. 31) è ïîñòðîèòü ýïþðû ïîïåðå÷íûõ è íîðìàëüíûõ ñèë è èçãèáàþùèõ ìîìåíòîâ.



Ðèñ 31

Ð å ø å í è å. 1. Ïðèíèìàåì îñíîâíóþ è ýêâèâàëåíòíóþ ñèñòåìû áàëêè âðåçàíèåì äîïîëíèòåëüíûõ øàðíèðîâ è çàìåíîé îòáðîøåííûõ ñâÿçåé îïîðíûìè èçãèáàþùèìè ìîìåíòàìè (Ðèñ. 32).



Ðèñ 32

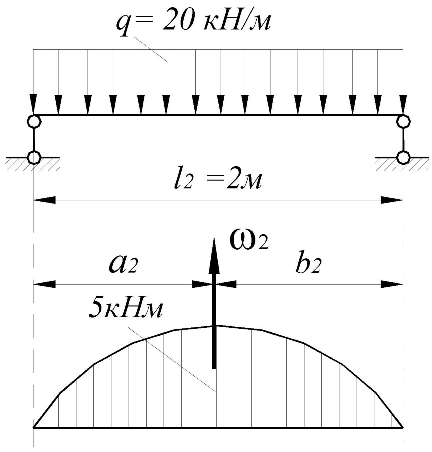
2. Ïî ÷èñëó ëèøíèõ îïîð îïðåäåëÿåì ñòåïåíü ñòàòè÷åñêîé íåîïðåäåëèìîñòè: n = 1.

3. Äëÿ êàæäîãî ïðîëåòà êàê äëÿ ñòàòè÷åñêè îïðåäåëèìîé áàëêè ñòðîèì ýïþðû èçãèáàþùèõ ìîìåíòîâ. Îïðåäåëÿåì âåëè÷èíû ïëîùàäåé ýïþð è êîîðäèíàòû èõ öåíòðîâ òÿæåñòè.

Ïðîëåò äëèíîé l_1: òàê êàê íà íåì íåò âíåøíèõ íàãðóçîê, òî

$$w_1 = 0,\ l_1 = 0,\ a_1 = 0,\ b_1 = 0.$$

Ïðîëåò äëèíîé l_2 (Ðèñ. 33):



Ðèñ 33

w_2 = (ql^3/12) = 20×2^3/12 = (40/3) êÍì^2 ,à$_2$ = 1ì; b_2 = 1 ì.

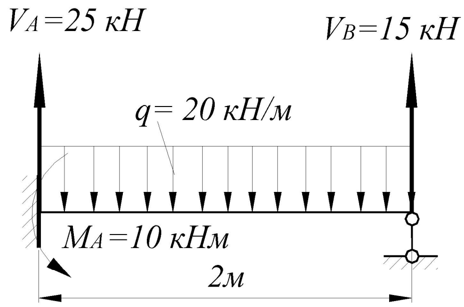
Ñîñòàâëÿåì óðàâíåíèå òðåõ ìîìåíòîâ [ñì. ôîðìóëó (4)]:

Ì$_0$$l_1$ + 2M_1(l_1 + l_2) + $M_2$$l_2$ = – 6[($w_1$$a_1$/$l_1$) + ($w_2$$a_2$ /l_2)]. Î÷åâèäíî, ÷òî

Ì$_0$ = 0 è Ì$_2$ = 0, ñëåäîâàòåëüíî, 2Ì$_1$×2 = – 6(40×1)/(3×2) = – 40, òîãäà

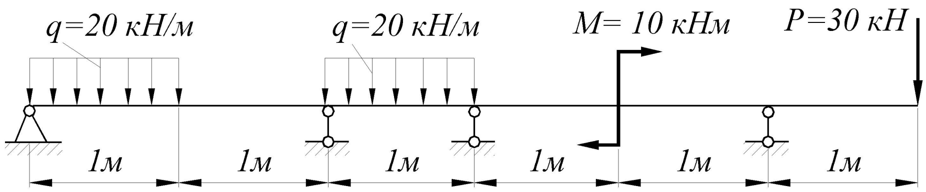
4Ì$_1$ = – 40; Ì$_1$ = –10 êÍì.

Òàê êàê îïîðíûé ìîìåíò ïîëó÷èëè ñî çíàêîì «ìèíóñ», ìåíÿåì ïðèíÿòîå íàïðàâëåíèå íà ïðîòèâîïîëîæíîå (Ðèñ. 34) è îïðåäåëÿåì âåëè÷èíó îïîðíûõ ðåàêöèé.



5. Ñòðîèì ýïþðû Q è Ì

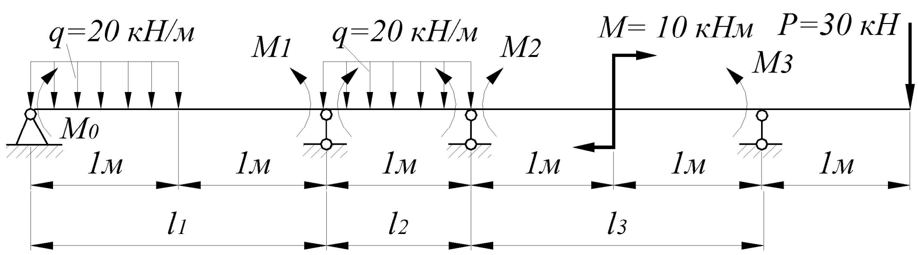
Ïðèìåð 4. Ðàñêðûòü ñòàòè÷åñêóþ íåîïðåäåëèìîñòü çàäàííîé áàëêè (Ðèñ. 36) è ïîñòðîèòü ýïþðû Q è Ì.



Ðèñ. 36

Ð å ø å í è å. 1. Îïðåäåëÿåì ñòåïåíü ñòàòè÷åñêîé íåîïðåäåëèìîñòè ïî ÷èñëó ëèøíèõ îïîð: n = 2.

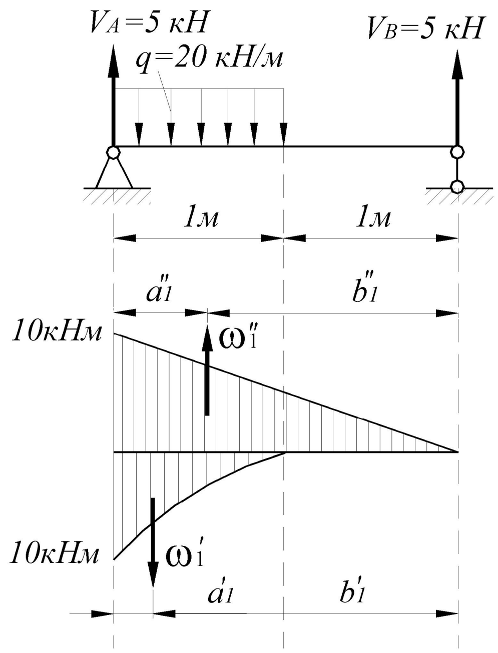
2. Ïðèíèìàåì îñíîâíóþ è ýêâèâàëåíòíóþ ñèñòåìû áàëêè âðåçàíèåì äîïîëíèòåëüíûõ øàðíèðîâ è çàìåíîé îòáðîøåííûõ ñâÿçåé îïîðíûìè èçãèáàþùèìè ìîìåíòàìè (Ðèñ. 37).



Ðèñ. 37

3. Äëÿ êàæäîãî ïðîëåòà êàê äëÿ ñòàòè÷åñêè íåîïðåäåëèìîé áàëêè ñòðîèì ýïþðû èçãèáàþùèõ ìîìåíòîâ. Îïðåäåëÿåì âåëè÷èíû ïëîùàäåé ýïþð è êîîðäèíàòû èõ öåíòðîâ òÿæåñòè.

Ïðîëåò äëèíîé l_1 (Ðèñ. 38):

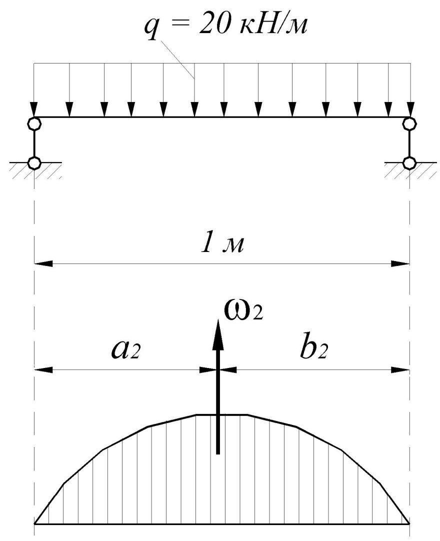


Ðèñ 38

w_1' = – (1/3)×10×1 = – (10/3) êÍ ì^2; w_1'' = (1/2)×10×2 = 10 êÍ ì^2;

a_1' = (1/4)×1 = 1/4 ì; a_1'= (1/3)×2 = 2/3 ì; b_1' = 3/4 + 1 = 7/4 ì; b_1''= (2/3)×2 = 4/3 ì.

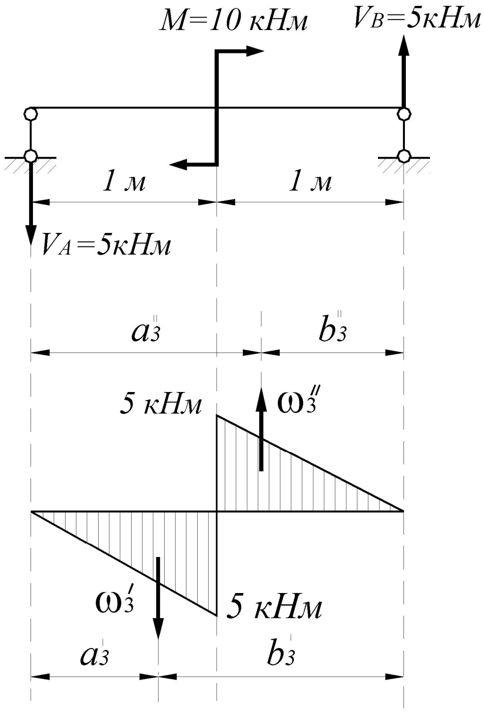
Ïðîëåò äëèíîé l_2 (Ðèñ. 39):



Ðèñ 39

w_2 = ql^3/12 = 20×1^3/12 = 5/3 êÍ ì^2; à$_2$ = b$_2$ = 1/2 êÍ ì^2;

Ïðîëåò äëèíîé l_3 (Ðèñ. 40):



Ðèñ. 40

w$_3$' = – (1/2)×5×1 = – 5/2 êÍì^2; w$_3$'' = (1/2) 5×1 = 5/2 êÍì^2;

à$_3$' = 2/3 ì; à$_3$'' = (1/3) +1 = 4/3 ì; b$_3$' = 4/3 ì; b$_3$'' = 2/3 ì.

Ñîñòàâëÿåì óðàâíåíèÿ òðåõ ìîìåíòîâ, ÷èñëî êîòîðûõ ðàâíî ñòåïåíè ñòàòè÷åñêîé íåîïðåäåëèìîñòè.

Ì$_0$l$_1$ + 2M$_1$(l$_1$ + l$_2$) + M$_2$l$_2$ = – 6[(w$_1$'a$_1$'/l$_1$) + (w$_1$''a$_1$''/l$_1$) + (w$_2$a$_2$ /l$_2$)];

Ì$_1$l$_2$ + 2M$_2$(l$_2$ + l$_3$) + M$_3$l$_3$ = – 6[(w$_2$a$_2$ /l$_2$) + (w$_3$'b$_3$'/l$_3$) + (w$_3$''b$_3$''/l$_3$)].

Î÷åâèäíî, ÷òî Ì$_0$ = 0, à Ì$_3$ = – Ð×1 = – 30 êÍì. Ïîäñòàâëÿåì â óðàâíåíèÿ ÷èñëîâûå çíà÷åíèÿ:

2Ì$_1$×3 + Ì$_2$×1 = – 6[– (10×1)/(3×4×2) + (10×2)/(3×2) + (5×1)/(3×2×1)];

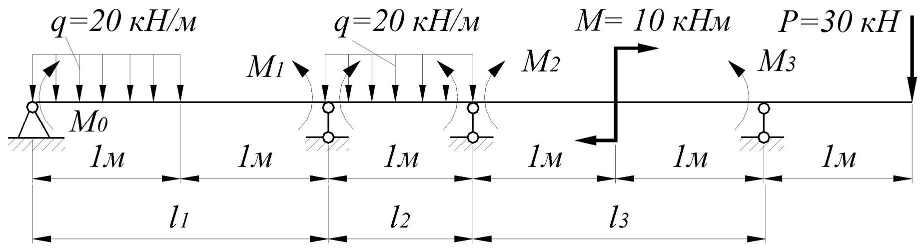
Ì$_1$ + 2Ì$_à$×3 – 60 = – 6[(5×1)/(3×2×1) – (5×1)/(2×3×2) + (5×1)/(2×3×2)], èëè

6Ì$_1$ + Ì$_2$ + 22,5 = 0; Ì$_1$ + 6Ì$_2$ – 60,0 = 0.

Ïðèìå÷àíèå: *â ïåðâîì óðàâíåíèè âêëþ÷àþò ìîìåíòû, íà÷èíàÿ ñ Ì$_0$, à âî âòîðîì, íà÷èíàÿ ñ Ì$_1$.*

Ðåøàåì ñîâìåñòíî ïîëó÷åííûå óðàâíåíèÿ òðåõ ìîìåíòîâ è íàõîäèì çíà÷åíèÿ Ì$_1$ è Ì$_2$: Ì$_1$ = –5,57 êÍì; Ì$_2$ = 10,92 êÍì.

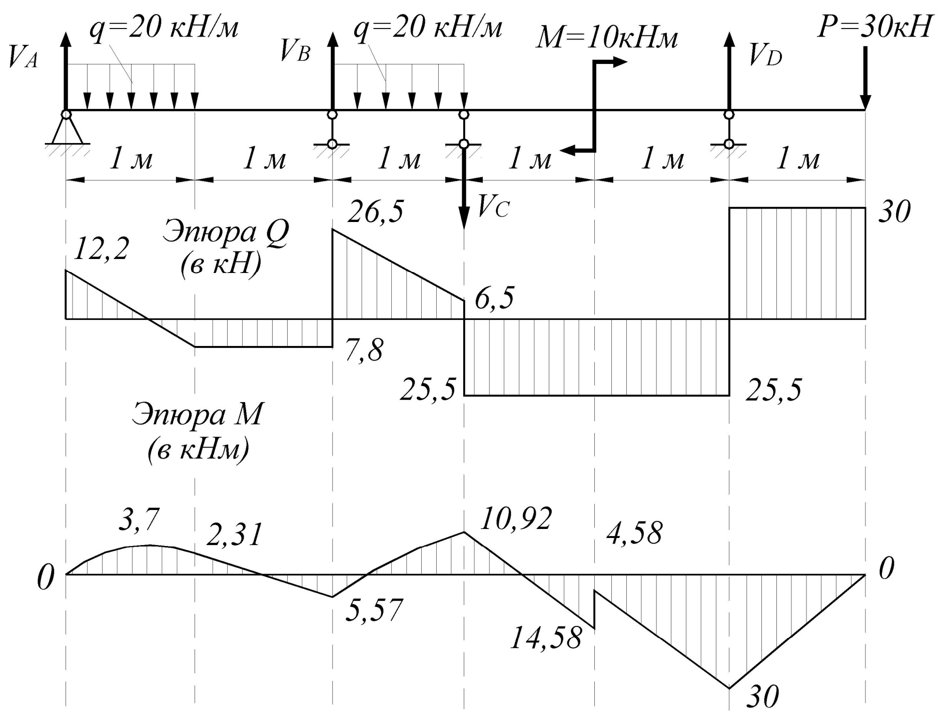
Íàïðàâëåíèå ìîìåíòà Ì$_1$ ìåíÿåì íà ïðîòèâîïîëîæíîå (Ðèñ. 41).



Ðèñ 41

Äëÿ êàæäîãî ïðîëåòà â îòäåëüíîñòè, êàê äëÿ ñòàòè÷åñêè îïðåäåëèìûõ

Ñâîäèì ýïþðû äëÿ âñåé áàëêè (Ðèñ. 42).



Ðèñ. 42

Ïî ñêà÷êàì íà ýïþðå Q îïðåäåëÿåì çíà÷åíèÿ îïîðíûõ ðåàêöèé:

$V_{\text{À}}$ » 12 êÍ; $V_{\text{Â}}$ » 34 êÍ; $V_{\text{Ñ}}$ = 32 êÍ; V_D » 56 êÍ.

Ñòàòè÷åñêàÿ ïðîâåðêà:

SY = $V_A+V_B+V_C+V_D$ – q×1 – q×1 – P = 12,2 + 34,3 – 32 + 56,5 – 20 – 20 – 30 = 0.